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DISCRETISATION OF STRUCTURES IN THE STOCHASTIC FINITE ELEMENT METHOD

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ABSTRACT

This paper presents the *principles of discretisation* of randomly deformable bodies in the stochastic finite elements. By a *stochastic finite element* is understand a subarea of a body whose characteristic are treated as random variables and not as stochastic processes.

Basic theorems are provided to enable discretisation of the stochastic process which describes a random characteristic of a body on a sequence of random variables which are the local mean of field in the area of a finite element.

The work is illustrated with *examples of the discretisation* of a stochastic processes of the correlation function with exponential and power class, and the estimation of covariance matrix of the original and differential of Fourier's series.

1. INTRODUCTION

The characteristic of the structure are a vector random field in time-space. The coordinates of time-space: vector of location of the material point of the structure, and time are non-random parameters of the random field of the structure. Among random characteristics of the structure one can distinguish geometry, parameters or material functions, and boundary conditions. Since load depends on the kind and configuration of the structure, the structure's characteristics also include static and kinematic input functions.

Division of a randomly deformable structures into the Stochastic Finite Elements is one of the basic problems of the Stochastic Finite Element Method. The principles of the Stochastic Finite Element Method are described in papers [1,2].

Discretisation of structures realisation (one of its item) is analogous to widely recognized discretisation in the classical (deterministic) of Finite Element Method. It is reduced essentially to providing a way of approximation of dependent variables (displacements, stresses, etc.) in inside the set subareas in the function of respective dimensions in the nodes of these subareas.

Division of structures into stochastic finite elements is essentially a discretisation of a continuous random field which in the one-dimension case, is degenerated to the stochastic process.

This paper provides the principles of discretisation of random field of a structure in relation to non-random parameters, i.e. space-time coordinates. Continuity of the field in the space of random elementary events.

The discretisation of the continuous stochastic process was performed by substituting it for a sequence of random variables. These variables are local mean values of the process in the subareas of the structure.

2. A STOCHASTIC FINITE ELEMENT

Let's consider a vector stochastic process X(t) dependent on a scalar parameter t. With the use of this model it is possible to describe, for example, a random process of the characteristics of rod section $X(x) = \{ X_1(x), X_2(x), X_3(x), ... \} = \{$ area of cross-section, inertia moment, plasticity limit, ... $\}$ according to place x on the rod length. Also, load S(t) is a stochastic process in time t.

Discretisation of the continuous stochastic process X(t) will be performed by replacing it by a sequence of random variables

$$X_i \overset{df}{=} \frac{1}{2T_i} \int_{t_i-T_i}^{t_i+T_i} X(t)\, dt, \quad (i = 1, 2, \ldots) \tag{1}$$

on segments of length $2T_i$ and centre at point t_i. Variables (1) defined by Riemann's stochastic integral are local means of the process X(t) in intervals $[t_i-T_i, t_i+T_i]$. The continuous parameter t (continuous time) was reduced to discrete indices i (discrete time).

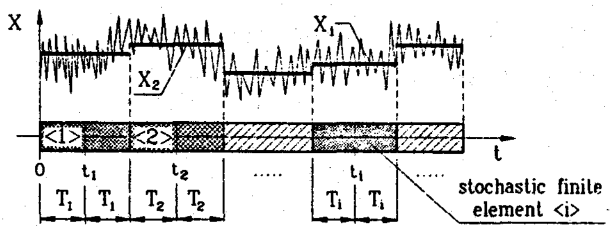

Fig.1 Realisation of discretised stochastic process

In Fig.1 is shown the realisation of a discretised scalar stochastic process X(t).

Segment $[t_i-T_i, t_i+T_i]$ of length $2T_i$ is one-dimensional **stochastic element** <i>, and variable X_i is a vector of the characteristics of this element. An element <i> is material if parameter t is a spatial coordinate. An element <i> is temporal if parameter t is time.

3. DISCRETISATION OF THE STOCHASTIC PROCESS

Below are given theorems concerning **discretisation** of a vector stochastic processes.

__Theorem 1__: *Expected value and covariance of the characteristics of stochastic processes*

Let $M_X(t) = E[X(t)]$, $R_{XX}(t_i, t_j) = E[X(t_i)X^T(t_j)]$ and $C_{XX}(t_i, t_j) = R_{XX}(t_i, t_j) - M_X(t_i)\, M_X^T(t_j)$ be trend, autocorrelation function, and autocovariance function of the stochastic process, respectively.

If random variables X_i and X_j are local mean values of this process on segments $2T_i$, $2T_j$:

$$X_i = \frac{1}{2T_i} \int_{t_i-T_i}^{t_i+T_i} X(t)\, dt \quad \text{and} \quad X_j = \frac{1}{2T_j} \int_{t_j-T_j}^{t_j+T_j} X(t)\, dt, \tag{2}$$

then: 1) expected value M_k of variable X_k $(k = i,j)$ is

$$M_k = E[X_k] = \frac{1}{2T_k} \int_{t_k-T_k}^{t_k+T_k} M_X(\tau)\, d\tau, \tag{3}$$

2) covariance of variables X_i and X_j is

$$Cov[X_i, X_j] = \frac{1}{4T_iT_j} \int_{t_i-T_i}^{t_i+T_i} \int_{t_j-T_j}^{t_j+T_j} C_{XX}(\tau_i, \tau_j)\, d\tau_i\, d\tau_j. \tag{4}$$

Proof:

1. $M_k = E[X_k] = E\left[\frac{1}{2T_i}\int_{2T_i} X(\tau)\,d\tau\right] = \frac{1}{2T_i}\int_{2T_i} EX(\tau)\,d\tau = \frac{1}{2T_i}\int_{2T_i} M_X(\tau)\,d\tau.$

2. Let's note, that $Cov[X_i,X_j] = E[X_i X_j^T] - E[X_i]E[X_j]^T$, whereas the expected value of the product of variables is

$$E[X_i X_j^T] = \left[\frac{1}{2T_i}\int_{2T_i} X(\tau)\,d\tau\right]\cdot\left[\frac{1}{2T_j}\int_{2T_j} X(\tau)\,d\tau\right]^T =$$

$$= \frac{1}{4T_iT_j}\int_{2T_i}\int_{2T_j} E[X(\tau_i)X^T(\tau_j)]\,d\tau_i\,d\tau_j = \frac{1}{4T_iT_j}\int_{2T_i}\int_{2T_j} R_{XX}(\tau_i,\tau_j)\,d\tau_i\,d\tau_j.$$

Remarks:

1. *Theorem 1* concerns homogeneous and *non-homogeneous* processes X(t).

2. If the characteristics of an element <i> are process X(t), and the characteristics of an element <j> are process Y(t), then covariance of mean X_i from process X(t) along the length of element <i> and mean X_j from process X(t) along the length of element <j>, will be calculated from (4) using the function of *mutual covariance* C_{XY} of these processes:

$$Cov[X_i,X_j] = \frac{1}{4T_iT_j}\int_{t_i-T_i}^{t_i+T_i}\int_{t_j-T_j}^{t_j+T_j} C_{XY}(\tau_i,\tau_j)\,d\tau_i\,d\tau_j. \tag{5}$$

Theorem 2: *Dependence between the covariance of stochastic elements and variances*

Let's make local means (1) X_i, X_j, X_o, X_{io}, X_{oj}, X_{ij} on segments $2T_i$, $2T_j$, $2T_o$, $2T_{io}$, $2T_{oj}$, $2T_{ij}$, respectively.

If these segments are determined by the nodes of stochastic elements in the way shown in Fig. 2, then there occurs:

$$Cov[X_i,X_j] = \frac{1}{2T_iT_j}\left\{T_0^2 Var[X_0] - T_{io}^2 Var[X_{io}] + T_{ij}^2 Var[X_{ij}] - T_{jo}^2 Var[X_{jo}]\right\} \tag{6}$$

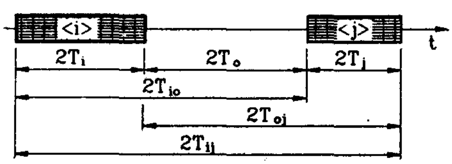

Fig.2. Denotes used in Theorem 2

Proof:

1. Let's note first that the following dependence occurs:

$$2(T_iX_i)(T_jX_j^T) = (T_oX_o)^2 - (T_{io}X_{io})^2 + (T_{ij}X_{ij})^2 - (T_{jo}X_{jo})^2.$$

This identity can be easily checked by substituting dependencies resulting fromthe properpies of integrals and Fig. 2:

$$T_{io}X_{io} = T_iX_i + T_oX_o, \quad T_{ij}X_{ij} = T_iX_i + T_jX_j + T_oX_o, \quad T_{oj}X_{oj} = T_jX_j + T_oX_o.$$

Also the following identity occurs for the expected values of local mean values:

$$2(T_iEX_i)(T_jEX_j)^T = (T_oEX_o)^2 - (T_{io}EX_{io})^2 + (T_{ij}EX_{ij})^2 - (T_{jo}EX_{jo})^2,$$

:n can be easily checked in analogous way.

2. It follows from the above identities that:

$$\mathrm{Cov}[X_i,X_j]= E[X_iX_j^T] - E[X_i]E[X_j]^T =$$

$$= E\left[\frac{1}{2T_iT_j} [(T_0X_0)^2-(T_{i0}X_{i0})^2+(T_{ij}X_{ij})^2-(T_{j0}X_{j0})^2]\right] -$$

$$-\left[\frac{1}{2T_iT_j} [(T_0EX_0)^2-(T_{i0}EX_{i0})^2+(T_{ij}EX_{ij})^2- (T_{j0}EX_{j0})^2]\right]=$$

$$= \frac{1}{2T_iT_j} \Big[T_0^2(E[X_0^2]-E[X_0]^2)-T_{i0}^2(E[X_{i0}^2]-E[X_{i0}]^2)+T_{ij}^2(E[X_{ij}^2]-E[X_{ij}]^2)-$$

$$- T_{j0}^2(E[X_{j0}^2]-E[X_{j0}]^2)\Big].$$

We get hence the thesis of Theorem 2.

Remark:

From Theorem 2 it is possible to calculate covariance between stochastic elements on the basis of variance in four areas containing these elements, and the segment contained between them. Tis property can be helpful in the estimation of covariance on the basis of the results of experimental measurements.

Theorem 3. *Covariance of a stationary discretised stochastic process*

Covariance of variables X_i and X_j defined in Theorem 1, obtained by averaging of the stationary process $X(t)$ is:

$$\mathrm{Cov}[X_i,X_j]= \frac{1}{4T_iT_j} \int_{\Delta t-\Sigma T}^{\Delta t+\Sigma T} a(\Delta\tau)\cdot C_{XX}(\Delta\tau)\, d(\Delta\tau) \ , \qquad (7)$$

where:

$$a(\Delta\tau)= \begin{cases} \Sigma T- \Delta T, & \text{if} \quad \Delta t-\Delta T< \Delta\tau <\Delta t+\Delta T, \\ \Sigma T- |\Delta\tau -\Delta t| & \text{in other cases.} \end{cases}$$

$$\Delta t= t_i-t_j, \quad \Sigma T= T_i+T_j, \quad \Delta T= |T_i-T_j|,$$

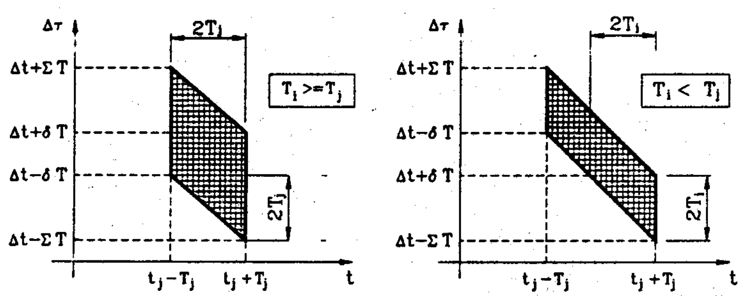

Fig.3. Integration area of the correlation function in Theorem 3

Proof:

For a stationary process , there occurs $C_{XX}(\tau_i,\tau_j) = C_{XX}(\tau_i-\tau_j) = C_{XX}(\Delta\tau)$. After substituting variables t_i for Δt in (7), we get:

$$Cov[X_i, X_j] = \frac{1}{4T_iT_j} \int_{t_j-T_j}^{t_j+T_j} \int_{t_i-T_i-\tau_j}^{t_i+T_i-\tau_j} C_{XX}(\Delta\tau)\, d(\Delta\tau)\, d\tau_j . \tag{8}$$

The above expression can be effectively integrated by τ_j. In Fig. 3a is represented the integration area in the case of $T_i >= T_j$, while in Fig. 3b - in the case $T_i < T_j$.

In the first case integration (8) can be rewritten in the following way:

$$4T_iT_j Cov[X_i, X_j] = \int_{\Delta t-\Sigma T}^{\Delta t-\delta T} \int_{t_j+T_j-(\Delta\tau-\Delta t+\Sigma T)}^{t_j+T_j} C_{XX}\, d(\Delta\tau)\, d\tau_j + \int_{\Delta t-\delta T}^{\Delta t+\delta T} \int_{t_j-T_j}^{t_j+T_j} C_{XX} d(\Delta\tau) d\tau_j$$

$$+ \int_{\Delta t-\Sigma T}^{\Delta t-\delta T} \int_{t_j+T_j-(\Delta\tau-\Delta t+\Sigma T)}^{t_j+T_j} C_{XX}\, d(\Delta\tau)\, d\tau_j = \int_{\Delta t-\Sigma T}^{\Delta t-\delta T} (\Delta\tau-\Delta t+\Sigma T)\, C_{XX}\, d(\Delta\tau) +$$

$$+ \int_{\Delta t-\delta T}^{\Delta t+\delta T} 2T_j C_{XX}\, d(\Delta\tau) + \int_{\Delta t+\delta T}^{\Delta t+\Sigma T} (-\Delta\tau+\Delta t+\Sigma T)\, C_{XX}\, d(\Delta\tau),$$

where: $\Delta t = t_i - t_j$, $\delta T = T_i - T_j$, $\Sigma T = T_i + T_j$.

In the case $T_i < T_j$ we shall obtain a similar result if instead of δT we shall substitute $\Delta T = |T_i - T_j|$, and instead of $2T_j$ we shall substitute $\min(2T_i, 2T_j)$. Taking into consideration relations between Δt and $\Delta\tau$ in particular integration intervals, and identity $\min(2T_i, 2T_j) = \Sigma T - \Delta T$, we get therefore the thesis of *Theorem 3*.

Remarks:

1. In *Theorem 3* the process $X(t)$ must be stationary, but its *ergodicity is not required.* This theorem *generalizes* known results for variance of the mean time of stationary process [5,6]. This concerns covariance of variables (3) averaged on segments of *different length.*

2. Variable Δt is a distance between stochastic element centres and substitutes distance $\Delta\tau$ between points of continuous process.

3. If $i \neq j$, then $|\Delta t| = \Sigma T$ (concerning covariance).

If $i = j$, then $\Delta t = 0$ (concerning variance).

Conclusions:

1. If the discretisations of the stationary stochastic process are performed on elements of equal length ($2T_i = 2T_j = 2T$), then it follows from *Theorem 3* that

$$Cov[X_i, X_j] = \frac{1}{2T} \int_{\Delta t-2T}^{\Delta t+2T} \left[1 - \frac{|\Delta\tau - \Delta t|}{2T}\right] C(\Delta\tau) d\Delta\tau \tag{9}$$

In the case of successive numeration of elements arranged on a line we have: $\Delta t = 2T(i-j)$, where $(i-j)$ is the difference of element numbers.

2. Variances of the mean local X_i will be obtained after substituting $\Delta t = 0$ in formula (7) and taking into account the evenness of autocovariance $C_{XX}(\Delta\tau) = C_{XX}(-\Delta\tau)$.

4. EXAMPLES

The following examples show the application of the theorem, proved to specification of covariance matrix between the characteristics of stochastic elements described by means of stationary stochastic processes with the correlation function exponential or power class.

Example 1: *Discretisation of the stationary process in a homogeneous are from the correlation function of exponential class.*

Let's consider discretisation of stochastic processes with a correlation function given by the formula

$$C(\Delta\tau) = \exp(-A|\Delta\tau|^{\alpha}) \tag{10}$$

where $A > 0$, $\alpha = 1,2$. Dependence (10) describes exponential class of the correlation function of random stationary fields in a wider sense [4].

Because the fluctuation scale θ [3]

$$\theta = \int_{-\infty}^{\infty} C(\Delta\tau)\, d\Delta\tau = 2\int_{0}^{\infty} \exp(-A|\Delta\tau|^{\alpha})d\Delta\tau = \begin{cases} -2/A & \text{for } \alpha=1, \\ \sqrt{\pi/A} & \text{for } \alpha=2, \end{cases} < \infty$$

(is finite), then process from the correlation function (10) can be ergodic (it is possible to show, that it is the case).

From formula (7) we have:

$$\mathrm{Cov}[X_i, X_j] = \frac{1}{4T_i T_j} \int_{\Delta t - \Sigma T}^{\Delta t + \Sigma T} a(\Delta\tau) \exp(-A|\Delta\tau|^{\alpha})d\Delta\tau. \tag{11}$$

where:

$$a(\Delta\tau) = \begin{cases} \Sigma T - \Delta T, & \text{if } \Delta t - \Delta T < \Delta\tau < \Delta t + \Delta T, \\ \Sigma T - |\Delta\tau - \Delta t| & \text{in other cases.} \end{cases}$$

If $\alpha = 2$, then integral (11) is not expressed by analytical functions, and it should be determined numerically.

If $\alpha = 1$, then after integration described in (11), we get

$$\mathrm{Cov}[X_i, X_j] = \frac{1}{2A^2 T_i T_j} [\cosh(A \cdot \Sigma T) - \cosh(A \cdot \Delta T)] \cdot \exp(-A \cdot \Delta t) \tag{12a}$$

$$\mathrm{Var}[X_i] = \frac{1}{2A^2 T_i^2} \left[2AT_i - 1 + \exp(-2AT_i) \right] \tag{12b}$$

where: $\Delta t = t_i - t_j$, $\Sigma T = T_i + T_j$, $\Delta T = |T_i - T_j|$.

From Theorem 2 we obtain an identical result.

Example 2: *Discretisation of the stationary process in a homogeneous are from the correlation function of power class.*

Let's consider discretisation of stochastic processes with a correlation function given by the formula

$$C(\Delta\tau) = (1 + A|\Delta\tau|^{\alpha})^{-1} \tag{13}$$

where $A > 0$, $\alpha = 1,2$. Dependence (13) describes a power class of the correlation function of random stationary fields in a wider sense [4].

The fluctuation scale θ [3] is

$$\theta = \int_{-\infty}^{\infty} C(\Delta\tau)\, d\Delta\tau = 2\int_{0}^{\infty} 1/(1+A|\Delta\tau|^{\alpha})\, d\Delta\tau = \begin{cases} \infty & \text{for } \alpha=1, \\ \pi/\sqrt{A} & \text{for } \alpha=2, \end{cases}$$

The process from the correlation function (13) is not ergodic for $\alpha=1$, and it can be ergodic for $\alpha=2$ (it is possible to show, that it is ergodic in essence).

Discretisation of the process will be performed by division of axis t into elements of equal length $2T_i = 2T_j = 2T$. From formula (7) we have

$$\mathrm{Cov}[X_i, X_j] = \frac{1}{2T} \int_{\Delta t-2T}^{\Delta t+2T} \left[1 - \frac{|\Delta\tau - \Delta t|}{2T}\right] / (1+A|\Delta\tau|^{\alpha})\, d\Delta\tau. \tag{14}$$

From (14), we get

$$\mathrm{Cov}[X_i, X_j] = \begin{cases} \frac{1}{4A^2T^2}\Big[(A\cdot\Delta t_1+1)\log(A\cdot\Delta t_1+1) - 2(A\cdot\Delta t+1)\log(A\cdot\Delta t_2+1) + \\ \quad (A\cdot\Delta t_2+1)\log(A\cdot\Delta t_2+1)\Big] & \text{for } \alpha=1, \\ \frac{1}{8AT^2}\Big\{ 2\sqrt{A}\cdot\Delta t_1 [\mathrm{arctg}(\sqrt{A}\cdot\Delta t_1) - \mathrm{arctg}(\sqrt{A}\cdot\Delta t)] + \\ \quad + 2\sqrt{A}\cdot\Delta t_2 [\mathrm{arctg}(\sqrt{A}\cdot\Delta t_2) - \mathrm{arctg}(\sqrt{A}\cdot\Delta t)] \\ \quad - \log(A\cdot\Delta t_1^2+1) + 2\log(A\cdot\Delta t^2+1) - \\ \quad - \log(A\cdot\Delta t_2^2+1) \Big\} & \text{for } \alpha=2. \end{cases} \tag{15a,b}$$

$$\mathrm{Var}[X_i, X_j] = \begin{cases} \frac{1}{2A^2T^2}\Big[(2AT+1)\log(2AT+1) - 2AT\Big] & \text{for } \alpha=1, \\ \frac{1}{4AT^2}\Big[4\sqrt{A}\cdot T\cdot \mathrm{arctg}(2\sqrt{A}\cdot T) - \log(4A\cdot T^2+1)\Big] & \text{for } \alpha=2. \end{cases} \tag{16a,b}$$

where: $\Delta t_1 = \Delta t - 2T$, $\Delta t_2 = \Delta t + 2T$,

Discretisation of **linearly** represented stochastic processes is shown in Example 3.

__Example 3__: *Covariance matrix of the original and differential of Fourier's series.*

The basic stochastic process $X_{<i>}$ in an element $<i>$ is given by formula:

$$X_{<i>}(\tau) = \sum_{i=1}^{n} [A_i \cos(\omega_i\tau) + B_i \sin(\omega_i\tau)], \tag{17}$$

where A_i, B_i are sequences of non-correlated random amplitudes of a mean equal to zero: $E[A_iA_j]=0$, $E[A_iB_j]=0$, $E[B_iB_j]=0$, $E[A_i]=0$, $E[B_i]=0$ and homogeneous variances $\mathrm{Var}[A_i] = E[A_i^2] = \mathrm{Var}[B_i] = E[B_i^2] = \sigma_i^2$. Frequencies ω_i are deterministic.

Trend and the autocorrelation function of process (17) is

$$M_{<i>} = 0\ , \quad R_{<i><i>} = E[X(\tau-\Delta\tau)X(\tau)] = \sum_{i=1}^{n} \sigma_i^2 \cos(\omega_i\Delta\tau), \tag{18}$$

so the process is stationary. Let's assume that in element $<j>$ is spread process $X_{<j>}$, which is the differential of process (17), i.e. linear operator $\Lambda = d/dt$. Thus we have:

$$X_{<j>}(\tau) = dX_{<i>}(\tau-\Delta t)/d\tau = \sum_{i=1}^{n} \omega_i \{-A_i \sin[\omega_i(\tau-\Delta t)] + B_i \cos[\omega_i(\tau-\Delta t)]\}, \tag{19}$$

It is possible to calculate that the expected value and the autocorrelation function of process $X_{<j>}$ is:

$$M_{<j>}=dM_{<i>}/d\tau= 0 \,, \tag{20}$$

$$R_{<i><j>}(\Delta\tau)=\sum_{i=1}^{n}\sigma_i^2 d[\cos[\omega_i(\tau_1-(\tau_2-\Delta t))]]/d\tau_2=\sum_{i=1}^{n}\omega_i\sigma_i^2\sin[\omega_i(\Delta\tau+\Delta t)], \tag{21}$$

$$R_{<j><j>}(\Delta\tau)=\sum_{i=1}^{n}\omega_i\sigma_i^2\, d[\sin(\omega_i(\tau_1-\Delta t-\tau_2+\Delta t)]/d\tau_1=\sum_{i=1}^{n}\omega_i^2\sigma_i^2\cos(\omega_i\Delta\tau). \tag{22}$$

Since correlation function (21) and (22) are stationary, then variances and covariances between local means, is possible to calculate using *Theorem 3.*

5. CONCLUDING REMARKS

This paper has provided the following **theorems** which enable discretisation of a body whose characteristics are described by a vector stochastic process: 1) with the expected value and covariance of the characteristics of stochastic elements, 2) with dependence between covariance of stochastic elements and variances, 3) with covariance of stationary discretised stochastic process.

Discretisation of a body into stochastic elements is in essence averaging of a stochastic process in the area of these elements, and changing it into a sequence of random variables.

Practical **application** of these theorems to the estimation of covariance matrix of the characteristics of the finite stochastic elements with a correlation function of exponential and power class and the estimation of covariance matrix of the original and differential of Fourier's series.

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STRESZCZENIE

W pracy podano zasady dyskretyzacji losowo odksztalcalnych cial na stochastyczne elementy skonczone. Przez stochastyczny element skonczony rozumie sie podobszar ciala, ktorego cechy traktowane sa jako zmienne losowe, a nie jako procesy stochastyczne.

Podano podstawowe twierdzenia umozliwiajace dyskretyzacje procesu stochastycznego na ciag zmiennych losowych, bedacych srednia lokalna pola w obszarze elementu skonczonego.

Prace opatrzono przykladami dyskretyzacji procesu stochastycznego o funkcji korelacji klasy wykladniczej i potegowej, a takze estymacji macierzy kowariancji oryginalu i rozniczki szeregu Fouriera.