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The Influence of Plastic Fronts on the Limit Load Capacity of Transversely Bended Beams in the Light of Experimental Investigations

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Summary—Analysis is made of the photomechanical tests of the propagation of plastic fronts in steel models of beams bended by a concentrated force. Occurence of elastic nucleus under concentrated load of the beam is shown, and the displacement of the critical section to the nucleus fringe is quantitatively presented. Finally the cause of occurence of capacities higher than theoretical is accounted for. Experimental results are compared with theoretical solutions.

NOTATION

N, V_N	average, coefficient of variability of isochromatic fringe order
N_{pl}	plastic isochromatic fringe order
F	load
F_{el}, F_{pl}	theoretical elastic, plastic load capacity
$F_{el,e}, F_{pl,e}$	experimental elastic, plastic load capacity
$L = 2l$	length of beam span
H	height of beam cross-section
$\lambda = l/H$	slenderness ratio of beam element
t	length of elastic nucleus of beam
C	height of elastic core of cross-section (or height of plastic zone plasticized by shear forces - in Table 1)
d	length of plastic zone
$\xi = x/l$	dimensionless coordinate of beam length

1. INTRODUCTION

Classical theoretical analysis of elasto-plastic beams and slabs [1,2] shows that plastic zones will occur in the limit plastic state in the form shown in Fig. 1. The accuracy of numerical methods (including finite element methods) has been evaluated by comparison with classical analytic solutions [3,4]. However, the analytical results were obtained after using many simplifying assumptions, which according to Hill [1] might have led to "approximations of indefinite accuracy".

Lack of a reliable experimental verification of classical solutions results in a hardly accurate interpretation of the results of numerical investigations. It was suggested in [4], for example, that the impossibility of obtaining full plasticization of the cross-section under a concentrated load results from a too small number of finite elements applied in the area of the neutral axis of the beam.

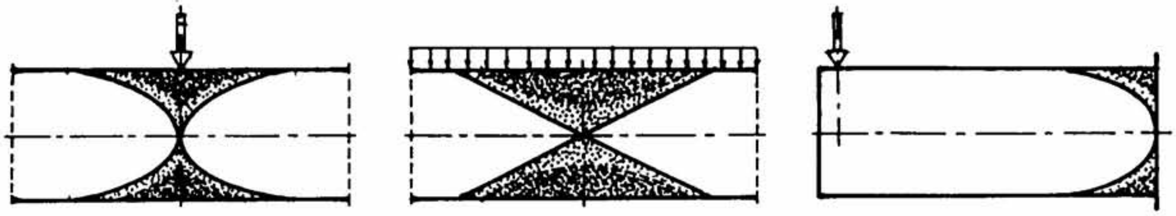


FIG.1. Classical shapes of plastic zones

In the present paper is made an analysis of the experimental investigations of the shapes of plastic fronts carried out in [5] on the steel models of beams bended transversely by concentrated forces. The aim of this analysis is to compare theoretical solutions with experimental results, verify computer algorithms applied to the determination of plastic zones, and to evaluate the shape of *plastic fronts* (lines reducing plastic zones) on the limit load capacity of transversely bended beams.

2. EXPERIMENTAL INVESTIGATION OF PROPAGATION OF PLASTIC FRONTS

In [5] are described photomechanical tests of the shape of plastic fronts carried out on 30 models (5 series containing 6 pieces each) of one-span beams of rectangular sections (Fig.2). The models of beams made of low mild steel were loaded by a concentrated force in midspan. The ratio of span length L to the height constant (H=40 mm) of the model cross-section was: $2\lambda = L/H = 4, 6, 8, 10, 12$.

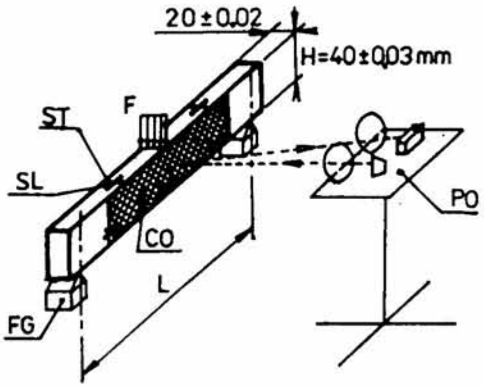


Fig.2. Scheme of the test stand

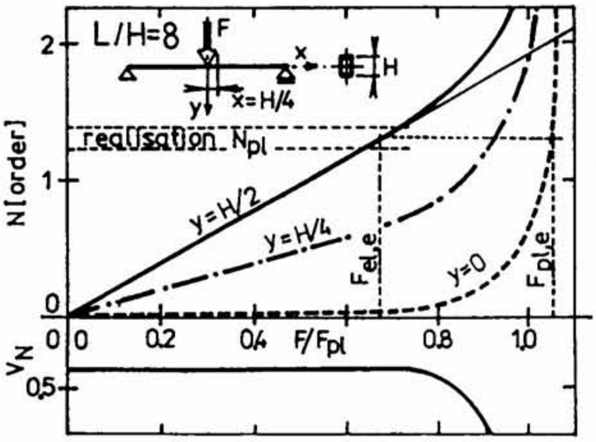


Fig.3. Estimation of the plastic isochromatic fringe order

Tests of the models were preceded by material examination. Longitudinal deformability of material (dependence of stresses on longitudinal strains) and transverse deformability (Poisson's coefficient in the elastic and plastic range) were determined. Average low yield point of material in seven samples was R=278.2 Mpa with a standard deviation 19.2 Mpa.

Loading of the models was accomplished by a hydraulic strentgh machine.

Simultaneity of the measurement of the loading force on force gauges <u>FG</u> (Fig.2), strains on longitudinal <u>SL</u> and transverse <u>ST</u> strain gauges , and coupling with photographying of the isochromatic fringe patterns in the cover <u>CO</u> made of epoxy resins was obtained by means of a measuring system SOLARTRON integrated with a minicomputer PDP-11-04. A portable reflective polariscope <u>PO</u> was used for the investigations.

In [5] are shown a number of photographs of isochromatic fringe patterns in an optically active cover, and a technique of estimating plastic fronts from the pictures of isochromatic lines.

Obtaining a plastic isochromatic fringe order was assumed to be a criterion of plasticization of the beam microstructure. *Plastic isochromatic fringe order* N_{pl} was estimated on the basis of the beginning of the nonlinearity of isochromatic order N measured at the lower edge of the beam in the function of load F (Fig.3).

Transformations of the isochromatic lines picture into relations F→N were made numerically by the method described in [5]. In Fig.3 are shown statistical parameters of those relations at selected points of a series of six beams L/H=8.

3. PROPAGATION OF PLASTIC FRONTS

In Fig.4 are shown the successive shapes of plastic fronts in a beam of the series L/H=4 for loads F=0.96 , 1.03 , 1.07 , 1.08 F_{pl}, where F_{pl} is a theoretical plastic load capacity of the beam. Plastic fronts of shapes shown in Fig. 4a, b, c, respectively, are formed with load F increasing over elastic load F_{el}=0.67 F_{pl} until a complete plasticization of the beam cross-sections at distance *t* from a concentrated force is achieved.

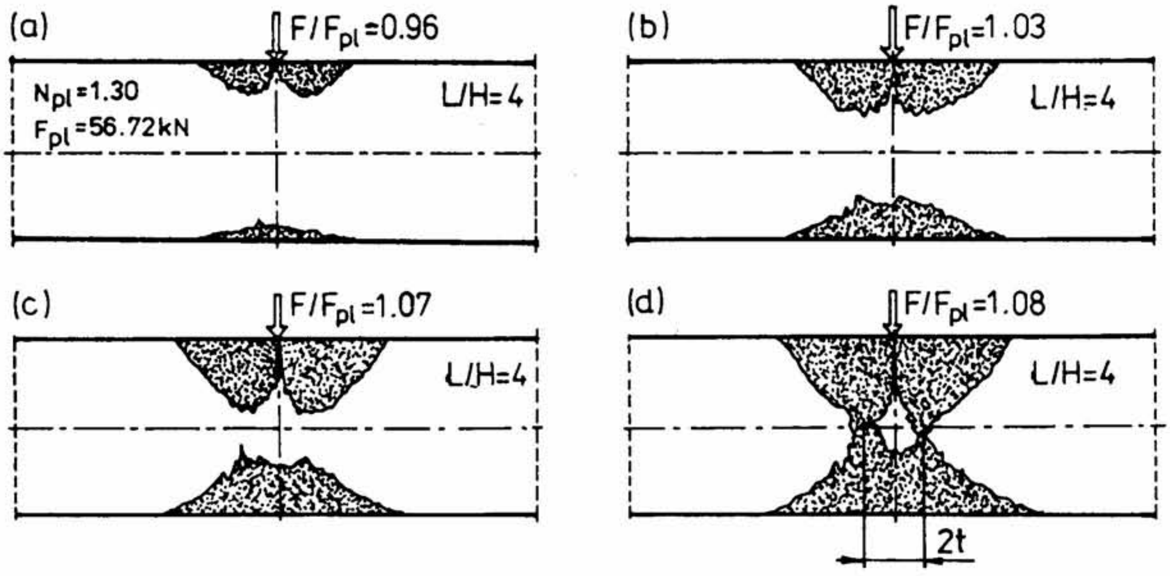


Fig.4. Shapes of plastic fronts in a beam of the series L/H=4

For a series of beams L/H=6, 8, 10 and 12, the shapes and propagation of plastic fronts is similar (Fig. 5). The numbers shown on the lines of plastic fronts are a ratio F/F_{pl}.

The most interesting feature of the plastic front is that *an elastic nucleus of the beam* is formed under a concentrated force and a complete plasticiztion of cross-sections occurs at distance l on both sides of the concentrated load.

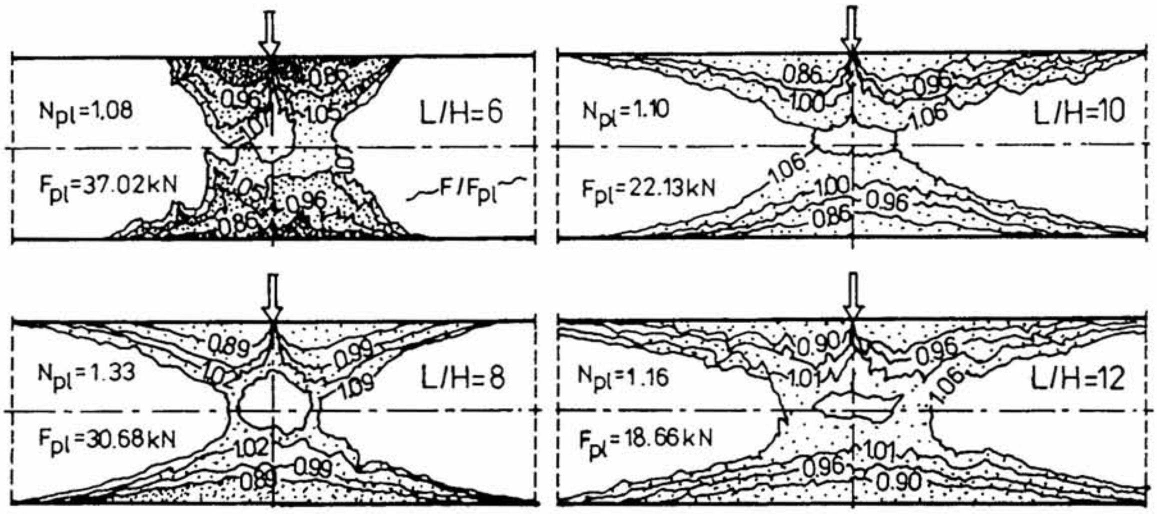


Fig. 5. Propagation of plastic fronts in beams of the series L/H=6,8,10,12

Realisations of plastic fronts (Fig. 5) have an irregular shape (jagged) due to the random shape and arrangement of the crystallites of real material.

The plastic fronts trend, determined on the basis of a set of realisations of plastic fronts (Fig. 6a), is smoothed (Fig. 6b).

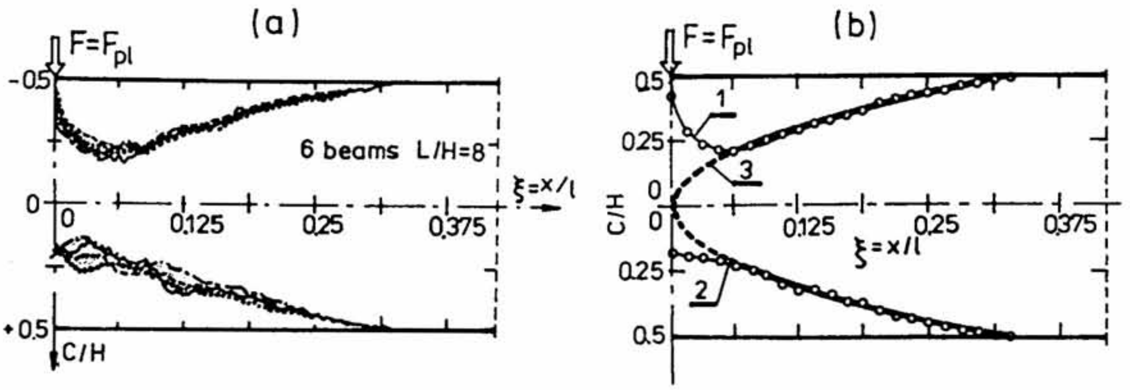


Fig. 6. Shape of plastic front in the theoretical plastic state of a series of beams L/H=8: (a) a set of realisations of plastic fronts, (b) approximation of plastic front trend.

A characteristic property of the front is also that its random variability decreases with regression from a concentrated force or with a decreasing cross-section effort.

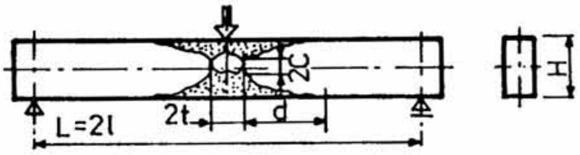
4. PARAMETERS OF THE GEOMETRY OF PLASTIC FRONTS AND LIMIT LOAD CAPACITY OF BEAMS

In Fig.6b is plotted a plastic front trend (circles) observed in the beams of the series L/H=8 subjected to load $F= F_{pl}$. The trend of lower and upper fronts can be described along the whole length of beam by polynomials (lines 1 and 2). Curves 1 and 2 are polynomials of the fifth order [5].

The results of approximation of only part of the plastic front at distance ≥ H/4 from the load axis are quite interesting. Regression curve 3 of equation $C/H=\sqrt{3}/2\cdot\sqrt{1.015 - 1.1018(1-\xi)}$ is almost identical with the theoretical equation $C/H=\sqrt{3}/2\cdot\sqrt{1 - 1(1-\xi)}$ derived from classical theories for load equal to theoretical plastic load capacity F_{pl} in spite of the fact that the beam's load capacity is not yet exhausted. Thus the beam has a higher load capacity that it results from the classical theory of plasticity.

In Table 1 are listed standard and average deviations in a series of six beams: limit elastic load capacity $F_{el,e}$, limit plastic load capacity $F_{pl,e}$,and also lengths of the elastic nucleus of beam 2t/H, height of the zone plasticized by shear forces 2C/H and lengths of plastic front d/l - observed in the limit plastic state.

TABLE 1. Limit load capacity of beams and parameters of the geometry of plastic fronts

				average ± standard deviation		
Series of beams	L/H	$\frac{F_{el,e}}{F_{pl}}$	$\frac{F_{pl,e}}{F_{pl}}$	$\frac{2t}{H}$	$\frac{2C}{H}$	$\frac{d}{l}$
A	4	0.68 ±0.02	1.068 ±0.036	0.402 ±0.008	0.377 ±0.036	0.34 ±0.01
B	6	0.69 ±0.02	1.073 ±0.019	0.490 ±0.031	0.275 ±0.008	0.30 ±0.03
C	8	0.66 ±0.01	1.063 ±0.035	0.563 ±0.024	0.200 ±0.004	0.34 ±0.02
D	10	0.67 ±0.01	1.064 ±0.026	0.618 ±0.032	0.161 ±0.010	0.34 ±0.01
E	12	0.68 ±0.01	1.050 ±0.027	0.621 ±0.045	0.131 ±0.007	0.33 ±0.01

5. CONCLUDING REMARKS

1) A theoretical estimation of limit load capacity and displacements of transversely bended beams on the basis of a plastic analysis of the structures shows considerable discrepancies from the experimental results.

2) The way of applying load has a significant influence on the mechanism of failure , shapes, and propagation of plastic fronts. In the case of concentrated application of external load an elastic nucleus of the beam is formed under a concentrated force.

3) Plasticization of the beam's cross-section occurs at the fringe of the elastic nucleus.

4) Occurrence of the elastic nucleus under a concentrated load (or reaction) causes an increase in the plastic load capacity of the beam bended transversely over a theoretical plastic load capacity.

5) In areas which are not included in the reducing influence of surface loads (outside the elastic nucleus of the beam) the expected plastic fronts are well described by the engineering theory of beam bending.

6) The method of finite elements can only be used to a visual demonstration of plastic zones in structural elements because they show imperfections close to the magnitude of a finite element. Determination of plastic fronts reducing plastic zones, obtained by this method, should be supported by experimental investigations.

7) The limit load capacity of beams bended transversely by a concentrated force can be determined from equilibrium equations on the basis of the theoretical load capacity cross-section F_{pl} under the condition of proper localization of plastic joints.

REFERENCES

1. R. HILL, *The mathematical theory of plasticity*, Oxford at the Clarendon Press , (1950).
2. A.A. ILYUSHIN, *Plasticite*, Editions Eyrolles, (1956).
3. G.C. NAYAK, O.C. ZIENKIEWICZ, Elasto-plastic Stress analysis. A Generalization for Various Constitutive Relations Including Strain Softening, *Int.J.Mech.Engng.*, V.5 ,(1972).
4. CZ. WOZNIAK , M. KLEIBER, *Nonlinear theory of structures* ,PWN, Warsaw-Poznan (1982), (in Polish).
5. L. CHODOR, Random load capacity of bending structures taking into consideration shear forces, Ph.D.Dissertation, *Proc. Depart. Building TU Wroclaw PRE 68/86*, Wroclaw (1986), (in Polish).
6. L. CHODOR, Z.KOWAL, Interaction of bending and shear of steel beams transversely bended in the light of experimental investigations, *Inzynieria i Budownictwo* (1988),(in Polish).