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NUMERICAL DETERMINATION OF ASYMPTOTIC REGRESSION CURVES OF COMPOSITE COMPRESSED BARS

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ABSTRACT: This paper present the results of an investigation of the equalisation of random results of the measurements of static equilibrium paths of composite compressed rods as a basis for the determination of their eigenvalues in the form of critical load bearing capacity. The critical load bearing capacity of composite rods was determined on the basis of equalised random measurements of the co-ordinates of static equilibrium paths *P(y)* of fixed compressed composite rods fixed on both sides, bearing random geometrical imperfections. For the equalisation of a few thousand random co-ordinates *(P,y)* is used as an asymptotic regression function on the basis of structural amplification of transverse displacements of compressed rods with initial buckling. The paper is illustrated with the equalisation of the results of the investigation of a series of models of composite compressed rods of nominal slenderness λ = 34; 80, and 180, reported in the paper. An algorithm of equalisation of the results of the investigation as well as conclusions resulting from the numerical analysis of result processing are provided .

1. INTRODUCTION

The experimental critical load bearing capacity of compressed rods is most frequently determined by Southwell's method [4] on rods fixed on hinged ends. The drawback of this method is a strong influence of small friction on hinges on critical load bearing capacity. The application of special permanent greases does not bring the results to reliable values. Kowal (1995) [4] derived physical relations which make it possible to extend the application of Southwell's method for the determination of critical load bearing capacity to models of rods fixed on both sides.

Southwell's method of determining critical load bearing capacity of rods with geometrical imperfections is based on the theoretical solution of a classical problem of nonlinear bending of initial flexure according to the sinusoid form

$$y_0 = A \sin(\pi x / l), \tag{1}$$

where: A - amplitude of rod flexure of length l, x - co-ordinate of rod axis.

In practice, compressed rods have geometrical imperfections of random shape and random displacement initial amplitude, which can be described by a Fourier series (2)

$$y = \sum_{i=1}^{n} A_i \sin(i\pi x / l), \tag{2}$$

It follows from research practice and theoretical investigations of initially bent rods, taking into consideration the effect of transverse flexures on effort (Bijak (1996) [1]) that at the beginning of load increment a change of the sign of the transverse displacement model. In such frequent cases during the further phase of load action, there follows an arrangement of the sign of transverse displacement. However, in the initial interval of load

$$P < P_{dn}, \tag{3}$$

rod transverse displacements behave contrary to Southwell's expectations. This problem has been known for a long time, but so far no practical method of experimental determination of the critical load bearing capacity of compressed rods has been developed. The behaviour of displacement in interval $P < P_{dn}$ can be theoretically predicted by measuring precisely geometrical imperfections and described by a Fourier series (2) determined individually for each investigation model. Differential equations from which we can determine individual critical load bearing capacity for each model has the form (4)

$$EIy^{IV} + P\left(y + \sum_{i=1}^{n} A_i \sin(i\pi x / l)\right)'' = 0, \tag{4}$$

where EI - stiffness of rod section.

Displacement amplification determined from differential equation (4) has a basic drawback in the case of rods built from fibrous composites: internal ("invisible") technological imperfections superimposed on "visible" geometrical imperfections cannot be initially measured. Occurrence of technological imperfections does not enable initial imperfections to be described by a Fourier series (4) in order to determine the structural formula for transverse displacement amplification.

After ordering the signs of increasing transverse displacement y there follows expected amplification which takes into account nonlinearity of second order, used by Southwell for the determination of the critical load bearing capacity of rods on the basis of their models with geometrical imperfections $y_0 = \sin \pi x / l$:

$$y = \frac{y_0}{1 - P/N_e}, \tag{5}$$

where: y - total transverse displacement, y_0 - initial rod flexure, P - axial load, N_e -Euler critical load bearing capacity of rod.

Currently, the measurements of co-ordinates (P_i, y_i) of static equilibrium path (SEP) are carried out automatically and registered digitally on diskettes in a set load step $\Delta P = P_i - P_{i-1}$, or better in a set displacement step $\Delta y = y_i - y_{i-1}$ The number of measurements (P_i, y_i) is practically arbitrary, most frequently being several thousand measurements on one model.

The measurements reflect random measurement errors which result from the testing machine, models, and measuring instruments. They have influence on transverse displacement amplification. The analysis of random displacements was carried out, amongst others, by Liaw et al. (1989) [6].

The use of Southwell's modified method [4] for experimental determination critical load bearing capacity of composite rods with internal and external imperfections requires application of a procedure which fulfils the following conditions: 1) we exclude the initial interval $0 < P < P_{dn}$ of the static equilibrium paths (SEPs) $P(y)$, 2) we assume occurrence of internal imperfections δ in a rod which give an external effect in displacements, 3) we take into account experiment errors y_e, 4) we equalise SEPs by the methods of probability calculus.

We know pairs (P, y) from measurements. However, we do not know initial flexure y_0 and internal eccentric cam δ. The critical load bearing capacity of initially bent metal rod [4,5] is determined directly on the basis of measured two coefficients (P_1,y_1) and (P_2,y_2) from the solution of the equation set (6):

$$N_{cr} = \frac{P_2 \cdot y_2 - P_1 \cdot y_1}{y_2 - y_1} \quad , \tag{6a}$$

$$y_0 = \frac{y_1 \cdot y_2 (P_2 - P_1)}{P_2 \cdot y_2 - P_1 \cdot y_1} , \tag{6b}$$

However, arbitrary adoption of points (P_i, y_i) $(i = 1,2)$ on the static equilibrium path gives different critical load bearing capacities N_{cr} due to the random properties of measurement.

The procedure of determining the critical load bearing capacity of compressed rods will be shown by the example of the investigation of the critical load bearing capacity of composite compressed rods carried out by Kowal and Gołaski (1996) [2] by means of a measurement of co-ordinates (P_i, y_i) static equilibrium paths written on diskettes.

The following characteristic co-ordinates can be distinguished in SEP:

1) Co-ordinate (P_{dn}, y_{dn}) which defines the end of the first interval $0 \le P \le P_{dn}$ of loads with strongly random transverse displacements in the respective interval $0 \le y \le y_{dn}$,

2) Co-ordinate (P_{up}, y_{up}) which defines interval $P_{dn} \le P \le P_{up}$ of the correct behaviour of transverse displacements $y_{dn} \le y \le y_{up}$ from interval $P > P_{up}$ of amplifying destruction or change of the internal structure of rod material;

3) Co-ordinates (P_L, y_L) of the limit load bearing capacity determined as the first maximum of equilibrium path.

Our task will be to equalise function $P(y)$ in order to determine the initial critical load bearing capacity of rod N_{cr} which has no change of the internal structure of rod material during axial loading.

Increase in reliability of the estimation of the critical load bearing capacity can be obtained by equalising the static equilibrium path $P(y)$ by the method of the least quadratic deviation in the area $P_{dn} \le P \le P_{up}$ Selection of the form of regression curve for the phenomenon of nonlinear transverse displacement of compressed rods with imperfections is of considerable importance. In engineering linear and nonlinear regressions are used. Nonlinear regression curves are usually adopted in the form of power polynomials

$$P = \sum_{i=0}^{i=n} a_i y_i^n \quad \text{or reverse function} \quad y = \sum_{i=0}^{i=n} a_i^* P_i^n \ ., \tag{7a,b}$$

Power polynomials are also used in Rene Thom's catastrophe theory for fictitious description of a catastrophe.

This paper proposes determination of the least critical load bearing capacity of compressed rods with geometrical imperfections by means of the nonlinear regression function conformable with the theoretical solution (4), henceforward referred to as an asymptotic regression function.

2. EQUALISATION OF THE RESULTS OF SEP MEASUREMENTS OF COMPRESSED RODS BY MEANS OF ASYMPTOTIC REGRESSION CURVES

Let us take into consideration an asymptotic regression curve in the form of (5) modified to the form of (8):

$$y = y_e + \frac{y_0}{1 - P/N_{cr}}, \tag{8}$$

where displacement of curve was made by value y_e which results from measurement errors (machines, sensors, record, etc.). This curve has asymptote $y \to \infty$ for $P = N_{cr}$

In order to estimate univocally parameters N_{cr}, y_0, y_e, it is necessary to know at least three measuring points (P_1, y_1), (P_2, y_2) and (P_3, y_3):

$$N_{cr} = \frac{-P_1P_2(y_2 - y_1) + P_1P_3(y_3 - y_1) - P_2P_3(y_3 - y_2)}{P_1(y_3 - y_2) - P_2(y_3 - y_1) + P_3(y_2 - y_1)}, \tag{9a}$$

$$y_0 = \frac{(y_2 - y_1)(y_3 - y_1)(y_3 - y_2)(P_2 - P_1)(P_3 - P_1)(P_3 - P_2)}{[y_1(P_3 - P_2) - y_2(P_3 - P_1) + y_3(P_3 - P_2)] \cdot [y_1(P_1P_3 - P_1P_2) + y_1(P_1P_2 - P_2P_3) + y_3(P_2P_3 - P_1P_3)]}, \tag{9b}$$

$$y_e = \frac{-y_1y_2(P_2 - P_1) + y_1y_3(P_3 - P_1) - y_2y_3(P_3 - P_2)}{y_1(P_3 - P_2) - y_2(P_3 - P_1) + y_3(P_2 - P_1)}, \tag{9c}$$

The example of curve (4) is represented in Fig. 1 with the marked adjustment interval. Interval $0 \le P \le P_{dn}$ of augmented measurement errors can be easily determined by the expert method discarding measurements which are strongly different from expected ones. The upper boundary P_{up} of interval $P_{dn} \le P \le P_{up}$ (or correspond $y_{dn} \le y \le y_{up}$) can be estimated experimentally by examining the structure of material or theoretically by examining deviations from critical load bearing capacity during the changes of the upper boundary by the trial method.

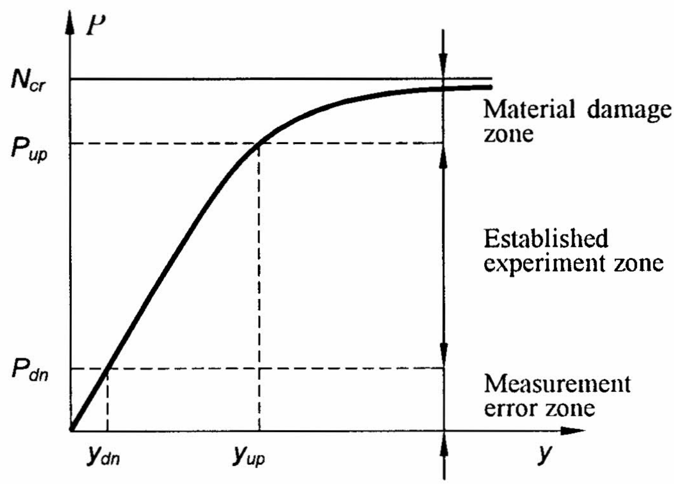

Fig. 1. Asymptotic regression curve

The program *Mathematica v.2.2.3* [8,9] was used for the determination of asymptotic regression. A procedure from a package *Statistics 'Nonlinear Fit'* of syntax

$$NonlinearFit[Data, ye + yo/(1 - P/Ncr), P, \{ye, yo, Ncr\}], \qquad (10)$$

where *Data* - a set of pairs of data *(y, P)*. Routines used by *NonlinearFit* locate local minimum in the χ^2 merit function, so careful choice of starting point be necessary. Starting points for the parameters are taken to be those values minimising χ^2 merit function out the set forming a 2^p factorial design based parameters range.
Apart from that was applied the program *MathCAD Plus 6.0 Profesional Edition* (Mathsoft, Inc (1986-1995)[7]. The following procedures were used: *Data Analysis/ Statistical function/Curve fitting.* From the procedures: *Linear regression, Polynomial regression, Multivariate polynomial regression, Linear combination of functions, Fitting arbitrary functions to data*, the last procedure was applied. Function *genfit(...)* was used to find coefficients of the given regression function for the set of experimental data.

Asymptotic regression curve (8) was adjusted to the results of the experimental investigation with a selection of adjustment selection $P_{dn} \le P \le P_{up}$ (or correspond $y_{dn} \le y \le y_{up}$). The beginning of adjustment interval was determined as $P_{dn} = k_{dn} P_{\max}$, where $P_{\max}$ the maximum on the equilibrium path measured during the investigation, and k_{dn} = 0,1 or 0.2; 0.3. The end of adjustment interval was selected according to dependence $y_{up} = k_{up} y_{\max}$, k_{up} = 1,00 to 0,20

In the prepared calculation sheets for the program Mathematica as well as MathCAD we obtained: 1) simple data statistics in the particular adjustment intervals (number of data, mean values of displacements and forces, standard deviations, and coefficients of linear correlation, 2) parameters of regression curve (8): initial displacements y_o and critical force N_{cr}, 3) plots of regression curves against experimental points.

The determination of the coefficients of asymptotic regression is shown by the examples from the work [2] in which the static equilibrium paths of composite rods of dimensions shown in Table 1 are determined experimentally.

Table 1. Measurement parameters [2] subject to equalisation

Item	Slenderness group	Sample symbol	Section bxh [mm]	Length l [mm]	λ	Number of measur.	N_{Lr} [kN] from measur.
1	λ_n=180	001	16,72x5,24	548	181,15	6969	0,981
2		003	16,39x5,18	557	186,75	1792	0,907
3		005a	16,67x5,17	528	176,84	889	1,030
4		006a	16,67x5,17	525	175,88	936	1,063
5	λ_n=80	007	16,72x5,24	255	84,29	1248	3,906
6		008	16,72x5,24	253	83,23	1432	4,004
7		009	16,72x5,24	236	78,01	3056	4,712
8		010	16,72x5,24	236	78,01	1400	4,688
9	λ_n=34	011	16,72x5,24	104,5	34,54	2559	19,849
10		012	16,72x5,24	104,5	34,54	2533	19,336
11		013	16,72x5,24	104,0	34,37	2719	20,361

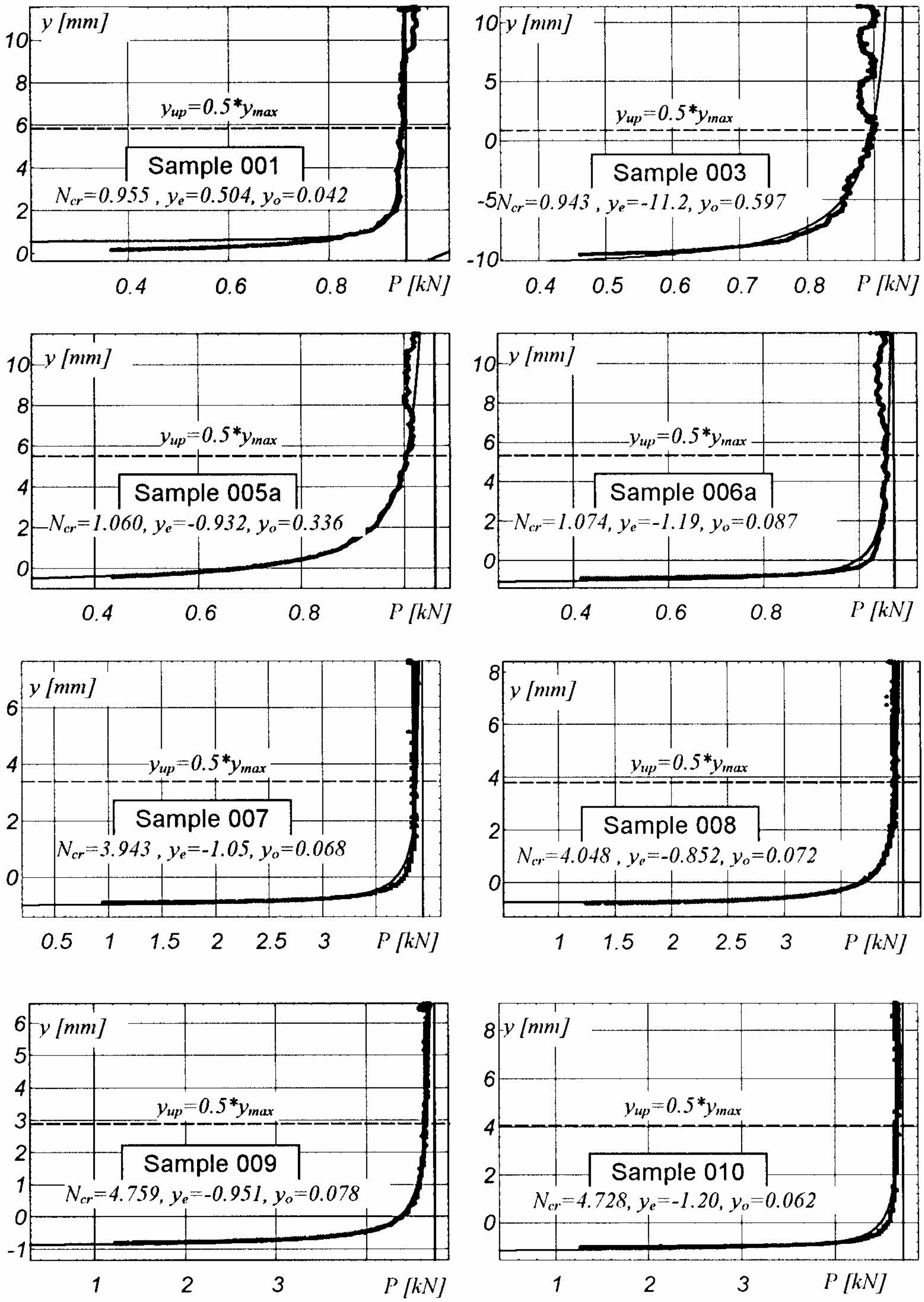

Fig.2. Exemplary regression curves

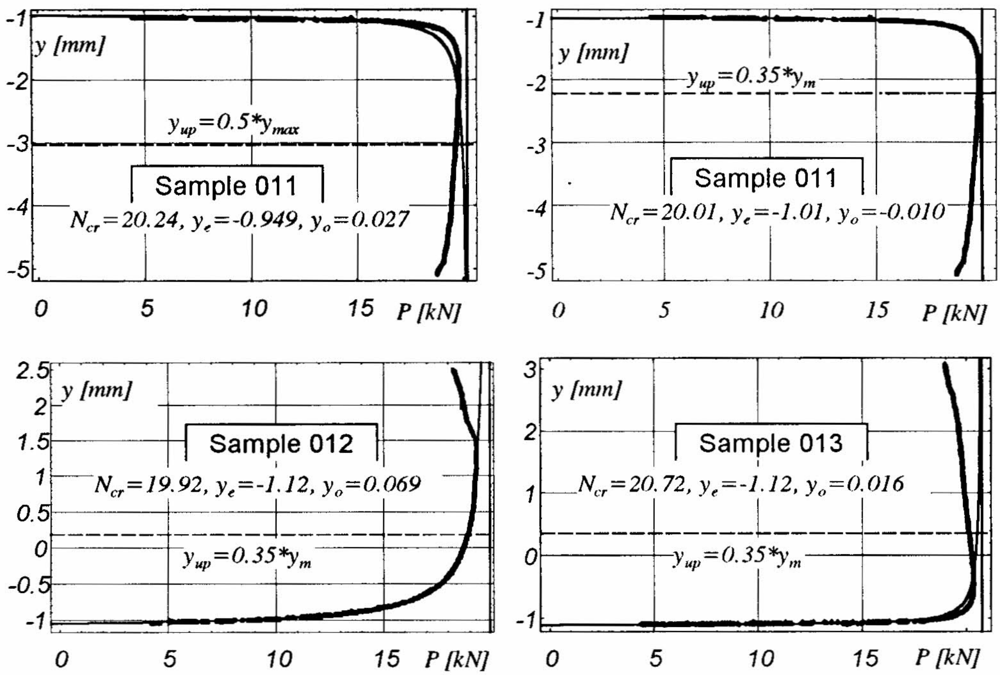

Fig.2. Exemplary regression curves (continued)

Table 2. Parameters of asymptotic regression curves

Item	Group	Sample symbol	N_L measur.	y_{up}/y_{max}	N_{cr}	y_0	y_e	N_{cr}/N_L	N_e [2]
			kN	-	kN	mm	mm	-	kN
1	λ_n=180	001	0,981	0,5	0,955	0,042	0,504	0,973	1,016
2		003	0,907	0,5	0,943	0,597	-11,2	1.040	0,838
3		005a	1,030	0,5	1,060	0,336	-0,932	1,029	1,029
4		006a	1,063	0,5	1,074	0,087	-1,19	1,010	1,020
5	λ_n=80	007	3,906	0,5	3,943	0,068	-1,05	1,009	3,959
6		008	4,004	0,5	4,048	0,072	-0,852	1,011	3,915
7		009	4,712	0,5	4,759	0,078	-0,951	1,010	4,787
8		010	4,688	0,5	4,728	0,062	-1,20	1,009	4,861
9	λn=34	011	19,849	0,35	20,01	-0,010	-1,01	1,008	27,797
10		012	19,336	0,35	19.92	0,069	-1,12	1,030	19,984
11		013	20,361	0,35	20,72	0,016	-1,12	1,018	29,368

After a few tests the beginning of adjustment interval was established arbitrarily as $P_{dn} = 0{,}2\,N_{\max}$.

Selection of the end of the adjustment interval has a significant influence on the estimation of critical force. After carrying out many analyses, stabilities of regression curves were determined.

On this basis the excess of co-ordinates occurring in irregular clusters was removed, and records bearing material damages were eliminated, too A correct estimation of load interval $P_{dn} \le P \le P_{up}$ and displacement $y_{dn} \le y \le y_{up}$ enabled us to adjust regression curves comfortably to the measured static equilibrium paths and to determine stable parameters N_{cr}, y_0, y_e. Stability coefficient was ca 1%.

Fig. 2 shows adjustment of regression curves measured SEP of composite compressed rods.

After a few tests the beginning of adjustment interval was established arbitrarily as $P_{dn} = 0{,}2N_{\max}$. tinuous line is superimposed on a selected regression curve together with its asymptote. Analyses performed are shown in Table 2. In Table 2 are shown critical load bearing capacities N_{cr} equalised from regression curves (8) and compared with Southwell's critical load capacities N_e experimentally determined in the work [2].

3. REMARKS AND CONCLUSIONS

Numerical experiments connected with adjustment of the asymptotic regression curve to experimental random co-ordinates of static equilibrium paths point to a significant area of elastic behaviour of composite compressed rods which exceeds critical load. This distinguishes composite rods from metal rods.

Application of the asymptotic regression curve and developing conditions for its stability enables one to determine critical bearing capacities and initial flexures as well as eccentric cams of compressed rods made from composite material.

On the basis of experimentally determined critical load bearing capacity N_{cr}, it is possible to construct a curve of critical strength applied in designing and referred to as buckling curve $\sigma_{cr}(\lambda)$.

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