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PLASTIC FAILURE PROCESS OF BEAMS TAKING INTO CONSIDERATION SHEAR FORCES

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SYNOPSIS

A theoretical analysis of the failure process of beams is performed taking into consideration shear forces.

In the experimental investigation of the process of plasticization of transversely bended beams a photomechanical method is employed for the estimation of the shapes of plastic fronts and tensometric test is employed to estimate section efforts.

It is shown that, taking into consideration shear forces in the limit plastic state of beam made from an elastic-perfectly plastic material, full plastic hinges are formed simultaneously in all critical sections.

However, in the performance of beam made from a work-hardening material, practically there will be no equalization of effort and the degree of plasticization of critical sections.

INTRODUCTION

Plastic failure of beams protected against stability loss consists in formation of such a number of full plastic hinges that the structural system (or its part) becomes geometrically variable.

Plastic hinge is identified with the state of full plasticization of section as a result of plastic redistribution of stresses which are in equilibrium sectional forces.

It follows from the classical, generally accepted theory of limit load capacity [1] that, during an increase of the load of a beam made form elastic-perfectly plastic material, plastic hinges are formed successively in critical sections until beam load capacity is exhausted. Such an understanding of the process of beam failure in which there occurs a distinct sequence of the formation of plastic hinges is deeply rooted in the consciousness of investigators and engineers. Beam plastic failure is identified with the formation of the last plastic hinge.

The present paper performs an analysis of the process of beam failure taking into consideration shear forces. Results of the experimental investigation of process of failure of transversely bended beams are presented using the photomechanical method and also tensometric method

It is shown that full plastic hinges are formed simultaneously in all critical sections..

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In beams made from a work-hardening material, reaching uniform plasticization in critical sections practically impossible. Due to strains from pressure of surface forces, an elastic kernel remains under concentrated forces, and a full plasticization of sections occurs at a certain distance on both sides of the concentrated force.

THEORETICAL ANALYSIS OF BEAM FAILURE

A theoretical analysis of the process of beam failure was performed by the example of beams with rectangular section made from an elastic-perfectly plastic material (Prandtl material).

Idealization of beams was performed by dividing it into one-dimensional elastic-plastic finite elements defined in the space of sectional forces. Formulas of Navier and Zuravski are assumed to be pertinent for the description of normal σ_x and shear τ_{xy} stresses. Continuity of stresses along the beam length is secured in a parabolic distribution of shear stresses in the elastic kernel and nulling of these stresses in the plasticized part of the section. It results from the basic equations of the mechanics of a solid body that in order to fulfill the Navier's conditions of equilibrium, the matrix of stresses must be supplemented with stresses σ_y which result from variability of stresses τ_{xy} along the beam length, i.e. from sectional changes of shear forces or variability of the height of the elastic core. However, even in this case the conditions of compatibility for strain are not fulfilled.

A model with properties described above is shown in Fig. 1.

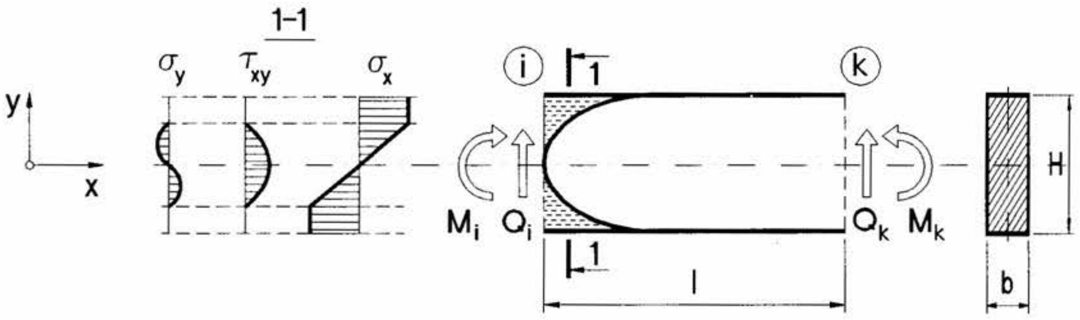


Fig. 1. One-dimensional model of an elastic-plastic beam element

In this model is omitted a stress from the pressure of surface forces, and on this account plastic front can be described by one formula along the whole length of the element, and critical section occurs in the place of the action of extreme bending moments.

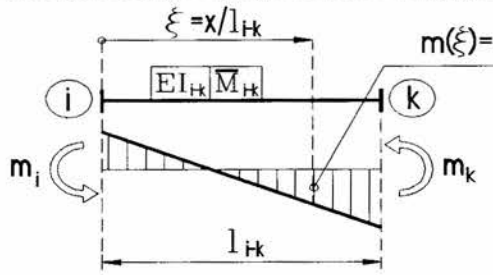
Stiffness of the elastic-plastic beam element was calculated from the energetic definition (Castigliano theorem - part II [2]):

$$\frac{dU}{dP_r} = Y_r \tag{1}$$

where: U - a complementary energy , P_r - fictitious generalized force, Y_r - displacement in the place and direction of the action of P_r.

In Table 1 are given analytical expression for a derivative of complementary energy U_{i-k} by a certain variable X for element *(i-k)* of a rectangular section unloaded between nodes. The boundary values of relative bending moments $m_x = M_x / \overline{M}_{i-k}$ $(x = i,k)$ depend on variable X and can adopt arbitrary values from the range $[-1, 1]$. Moment $\overline{M}_{i-k}$ is a pure plastic load capacity of the section (for rectangular section bxH is: $\overline{M}_{i-k} = f_y b H^2/4$, where f_y is a plasticity limit). Contribution of shear stresses to a derivative of energy is expressed by the components r_i , r_k .

TABLE 1. Derivation of complementary energy for an element of rectangular section

		$m(\xi) = m_i - \xi(m_i + m_k)$ $\frac{dU_{i-k}}{dX} = \mathbf{F}_{i\text{-}k}\ (m_i,\ m_k,\ l_{i\text{-}k}) =$ $= \overline{M}_{i\text{-}k}{}^2\ \frac{l_{i-k}}{EI_{i-k}}\ [\ \frac{dm_i}{dX}\ (t_i + r_i) + \frac{dm_k}{dX}\ (t_k + r_k)]$
1	$\lvert m_i \rvert <= 2/3$ $\lvert m_k \rvert <= 2/3$	$t_i = A[c_i - 27m_k^3/4]$, $r_i = 3B(m_i + m_k)$ $t_k = A[c_k - 27m_i^3/4]$, $r_k = 3B(m_i + m_k)$
2	$\lvert m_i \rvert <= 2/3$ $\lvert m_k \rvert >= 2/3$	$t_i = A[c_i + d_k - 27m_k]$, $r_i = B[g_k + 3(2m_i + m_k)/2]$ $t_k = A[d_k + e_k(m_i + m_k) - 27m_i(m_i^2/4 - 1)]$, $r_k = B[g_k + 3m_i/2 + f_k(m_i + m_k)]$
3	$\lvert m_i \rvert >= 2/3$ $\lvert m_k \rvert <= 2/3$	$t_i = A[d_i + e_i(m_i + m_k) - 27m_k(m_k^2/4 - 1)]$, $r_i = B[g_i + 3m_k/2 + f_i(m_i + m_k)]$ $t_k = A[c_k + d_i - 27m_i]$, $r_k = B[g_i + 3(2m_k + m_i)/2]$
4	$\lvert m_i \rvert >= 2/3$ $\lvert m_k \rvert >= 2/3$	$t_i = A[d_i + d_k + e_k(m_i + m_k)]$, $ri = B[\ gi + g_k + f_k(m_i + m_k)]$ $t_k = A[di + d_k + e_k(m_i + m_k)]$, $r_k = B[\ gi + g_k + f_k(m_i + m_k)]$
$A = \frac{2}{81(m_i + m_k)^2}$, $B = \frac{1+v}{15}(\frac{H_{i-k}}{l_{i-k}})^2$, $c_x = \frac{27}{4} m_x^2 (3m_{xx} + 2m_x)$, $d_x = [20 - 12\sqrt{3}(1 - \lvert m_x \rvert)^{3/2}]sgn(m_x)$ $e_x = -18\sqrt{3}(1 - \lvert m_x \rvert)^{1/2}$, $f_x = \frac{\sqrt{3}}{2\sqrt{1 - m_x}}$, $g_x = (2 - \sqrt{3}(1 - \lvert m_x \rvert)^{1/2} sgn(m_x)$, where: *(x = i,k)*, *(xx = k,i)*, *(E,v)* - (Young's Modulus, Poisson's ratio) in the elastic range		

If node sectional forces are combined in vector $\mathbf{M} = [M_i, M_k]^T$, then the corresponding node displacements of element (i-k) $\mathbf{Y} = [\phi_i, \phi_k]^T$ will be obtained from Table 1 after substituting $X = M_i$ or $X = M_k$, respectively. Because $dm_x/dM_x = 1/\overline{M}_{i-k}$ $(x = i,k)$, then

$$\mathbf{Y} = \overline{M}_{i\text{-}k}\ \frac{l_{i-k}}{EI_{i-k}}\ [(t_i + r_i),\ (t_k + r_k)]^T. \tag{2}$$

In the elastic range $\lvert m_i \rvert < 2/3$, $\lvert m_k \rvert < 2/3$, and after performing adequate transformation, one gets known linear relations $t_i = \frac{1}{6\overline{M_{i-k}}}(2M_i - M_k)$, $t_k = \frac{1}{6\overline{M_{i-k}}}(2M_k - M_i)$, which define the matrix of flexibility irrespective of the shape of cross-section. In other cases coefficients t_x, r_x are nonlinear functions of node moments.

By a limit plastic state will be regarded such a state in which plastic mechanism is put into operation, and displacements tend to infinity. It can be shown [3], that under the assumption of small displacements, plasticization of whole transverse section of beam determined statically, need not be unequivocal with reaching of the limit plastic state.

Without infringing the assumption of small strains, but taking into consideration shear forces, it is possible to show simultaneity of the formation of full plastic hinges in the limit plastic state. It will be shown by the example of a two-span beam loaded by concentrated forces in midspan.

In order to estimate the coefficients of section effort over support mx and under force m_P in the function of load P, the condition of kinematic admissible $dU/dm_x = 0$ use to one's adventage. Combining adequate components from Table 1 the following nonlinear equations was obtained

$$F(0,m_p,l) + F(-m_p,-m_x,l) = 0. \tag{3}$$

Relative moments m_p and m_x are connected with the condition of equilibrium $2m_p - m_x = Pl/\overline{M}$, which $\overline{M}$ is pure plastic load capacity of the section. The results the above equations are plotted in Fig. 2.

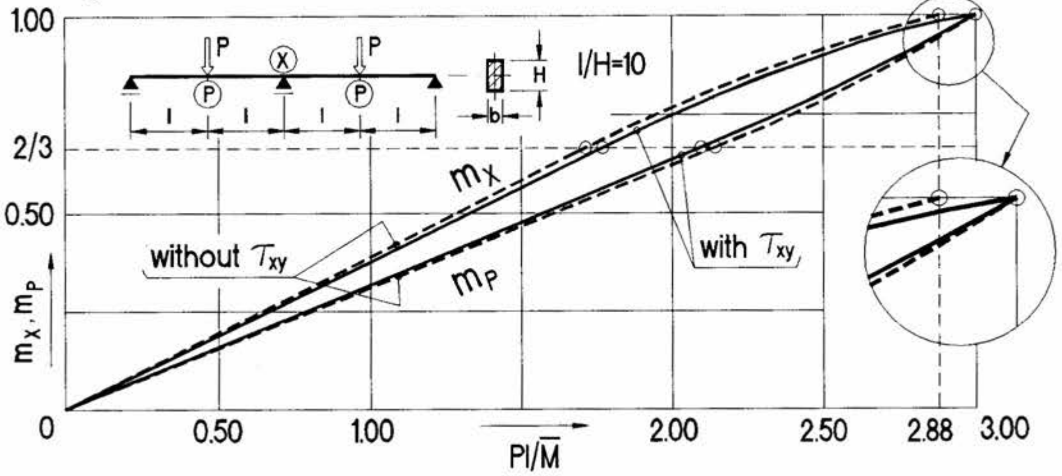


Fig.2. Example of simultaneity of the formation of plastic hinges

If we take into consideration shear forces, completion of the formation of plastic hinges in the span and over the support follows under the same load. It can be shown that they are full hinges, ie. they lead to infinite displacements of the beam axis.

If we neglect shear forces, the plastic hinge will be first formed overthe support under load $2.88\ Pl/\overline{M}$, and under the force already under the load of $3.00\ Pl/\overline{M}$.

Simultaneity of the formation of plastic hinges in the limit beam plastic state has important practical consequences. The beam static scheme is unchangeable until the moment of failure, and a change in the stiffness of sections in the process of plasticization is continuous and smooth.

THE PROCESS OF BEAM FAILURE IN THE LIGHT OF EXPERIMENTAL TESTS

An experimental photomechanical investigation of the shape of plastic fronts [4,5] was performed. In Fig.3 are shown plastic fronts in the realization of beam of rectangular section.

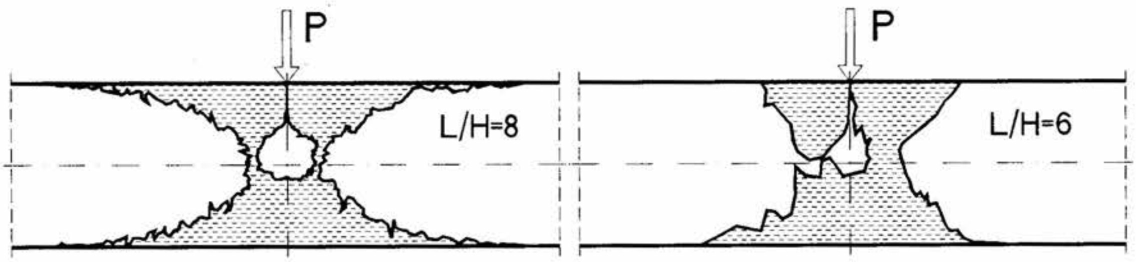


Fig.3. Shapes of plastic fronts in a beam of rectangular section [4]

It was observed systematically that the elastic kernel remained under a concentrated force, and a full plasticization of the section follows at distance t on both sides of the force. Realizations of plastic fronts have an irregular shape (rugged).

Apart from the photomechanical tests, tensometric measurements of strains were made, and also measurements of the reaction of two-span beams. The investigations performed on 2 series two-span beams (L/H = 6 and L/H = 10) with 5 beams in each series. For given load level 25 realizations of a given statical quantity were obtained. All these realizations constituted a basis for the calculation of the expected value and the coefficient of variation of sectional bending moment.

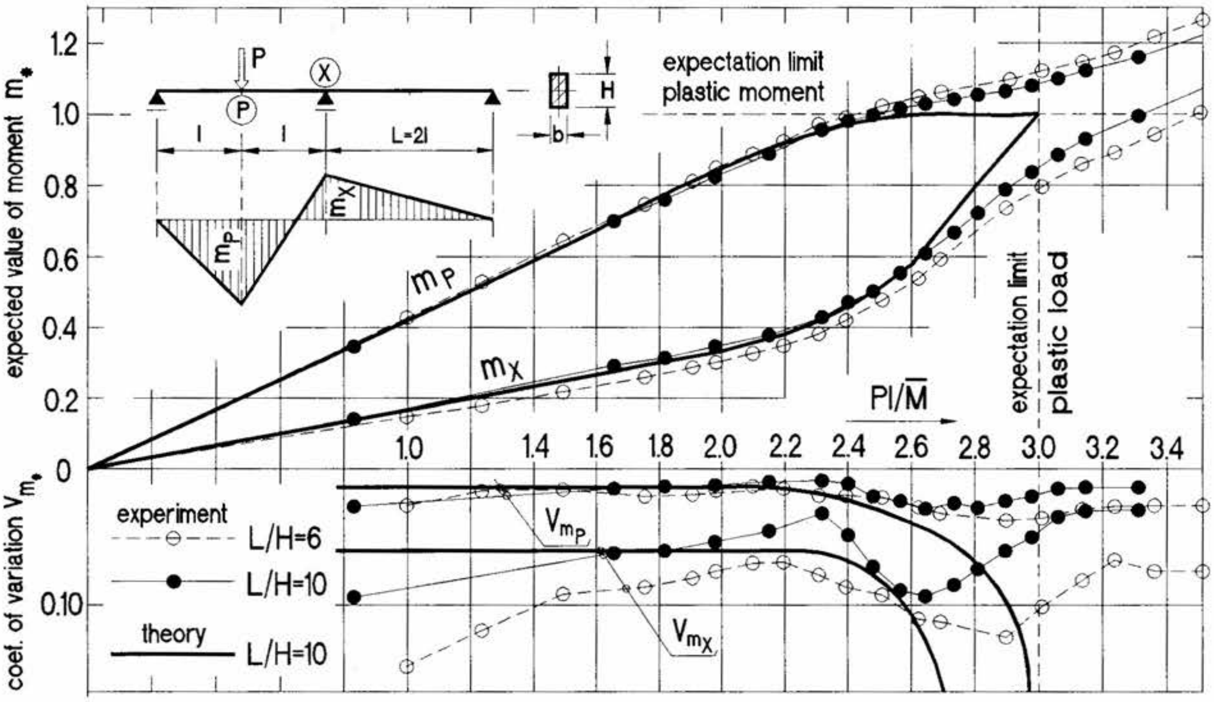


Fig.4. Trend and coefficient of variation of the relative span moment m_P, and support moment m_X two-span beam observed in the experiment and comparison with theory

In Fig. 4 is shown a process of the formation of plastic hinges in support and span section of series $L/H=6$ (broken line), and $L/H=10$ (continuous line with dots). A thick continuous line shows a theoretical dependence of effort m on load $Pl/\overline{M}$, calculated taking into consideration the random elasticity of supports. The theoretical values of random coefficients of variation were estimated using the Stochastic Finite Element Method [6].

For Prandtl's model of material full plasticization of all hinges, occurs at the same moment. However, in a real beam practically there will not occur equalization of the plasticization state of critical sections due to the work-hardening of material and the action of stresses from the pressure of surface forces. Support section is always more strained then span section.

CONCLUDING REMARKS

1. In beams made from elastic-perfectly plastic material full plastic hinges are formed simultaneously in all critical sections. This phenomenon can be presented theoretically after an improvement of the mathematical model of a beam by taking into consideration Shear forces. Shear forces considerably change the qualitative character of the failure process although quantitatively the effort changes are not high, particularly in beams slenderness encountered in practice.

2. In beams made from a work-hardening material under a monotonically increasing one-parameter load reaching equal effort (and plasticization) of critical sections is practically impossible. The criterion of a limit state of such beams must be formulated in rheological terms.

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