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DESIGN OF MULTI-SPAN CYLINDRICAL RESERVOIR WITH DEFORMABLE CROSS-SECTION

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1. Introduction

Techniques of designing horizontal cylindrical reservoirs recommended in manuals [1,2,3] are related as a rule to shells with non-deformable cross-sections.

In the case of reservoirs with high slenderness of the shell casing ($r/g > 175$, r- reservoir radius, g- thickness of the shell wall), the deformability of the cross-section exerts an essential influence on the shell forces in the membranous and bending state, particularly in the case of the lack of intersupport frames. Analytical solutions are used for the calculation of stresses or shell forces in a freely supported reservoirs stiffened over supports [1,3].

It results from the authors' tests that formulae given in textbooks [1,3] fail in the case of "short" cylinder spans ($l/r < 2$, l-span length). For such reservoirs series given in [1] to calculate shell forces are divergent, particularly due to a considerable influence of bending state.

The present paper presents the technique of calculation of multispan vertical reservoirs with highly deformable cross-section taking into account the bending state and local stability of the shell. The presented technique allows to perform cost-saving designs of vertical reservoirs because due to a reduction of the number of frames to an indispensable minimum. A characteristic feature of the proposed computer program is a short calculation time with a relatively high accuracy in comparison with programs of the finite element method.

The paper is illustrated by an example of a prototype project of a reservoir of capacity 200 m^3 for the production of biogas for agriculture.

2. Statics of multispan reservoirs with transverse load

2.1. Superposition of shell states

Solution of a multispan shell supported by infinitely stiff discs on supports was approximated by finding a solution for a span loaded transversely, and for a span loaded by a moment at a support (Fig. 1).

The support moments M were estimated for a continuous beam with a resultant transverse load q (beam load) around the shell perimeter. In the case of the most frequent transverse reservoir loads these resultants are: 1) specific weight $q=2\pi rg$, 2) liquid pressure $q=\pi r^2\gamma\ [1-(\varphi_0-\sin\varphi_0)/(2\pi)]$, 3) gas pressure $q=-2pr\sin\varphi$, 4) wind $q=0.799rp\ C\ \beta$,5) snow $q=2rs$ (Fig.2).

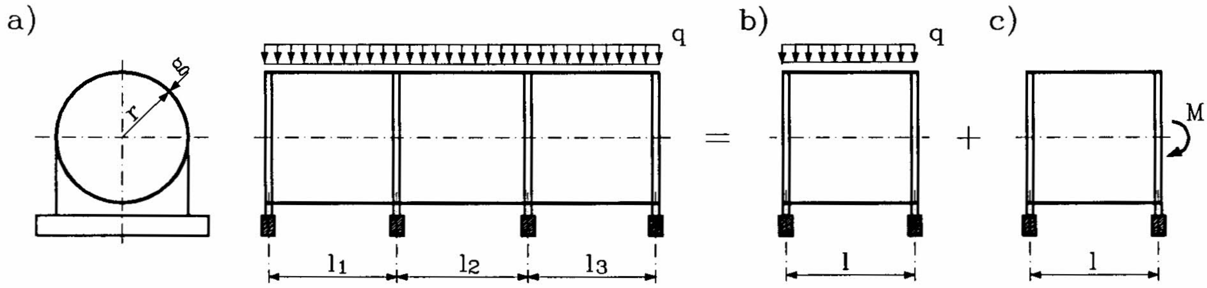

Fig. 1

Wind load of beams was obtained after integrating around the perimeter the values given in the polish standard.

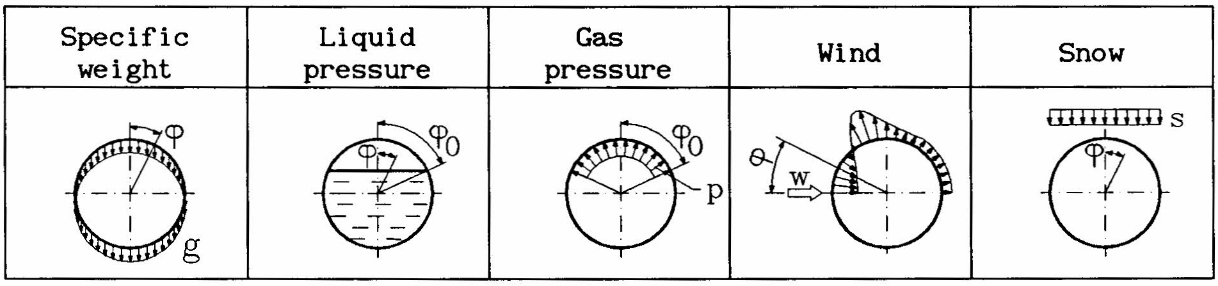

Fig. 2

2.2. Solution of the shell loaded by a moment at the support

Using the precise dependencies for short shells [5], a closed analytic expression was found for the displacements and shell forces of the antisymmetric state of the shell stiffened over the supports and loaded as shown in Fig. 3a.

After solving adequate differential equations, the following formulae for determination of radial displacements w, shell forces N_x, N_φ, $N_{x\varphi}$ and bending moments M_x, M_φ (Fig. 3b) were obtained.

$$w= - \frac{M}{\pi rEg}\left[[\nu-(l^2-x^2)/(6r^2)]x/l+\nu\frac{\bar{K}_2K_2-\bar{K}_1K_3}{\bar{K}_1\bar{K}_3-\bar{K}_2^2}\right]\cos\varphi, \tag{1a}$$

$$N_x=\frac{M\,x}{\pi r^2 l}\cos\varphi,\quad N_\varphi=-\frac{\nu M}{\pi r^2}\frac{\bar{K}_2K_2-\bar{K}_1K_3}{\bar{K}_1\bar{K}_3-\bar{K}_2^2}\cos\varphi,\quad N_{x\varphi}=\frac{M}{\pi rl}\sin\varphi, \tag{1b,c,d}$$

$$M_x=\frac{Mg}{\pi r^2}\frac{\nu}{4\sqrt{3}\sqrt{1-\nu^2}}\frac{\bar{K}_2K_0-\bar{K}_1K_1}{\bar{K}_1\bar{K}_3-\bar{K}_2^2}\cos\varphi,\quad M_\varphi=\frac{Mg}{\pi r^2}\frac{\nu^2}{4\sqrt{3}\sqrt{1-\nu^2}}\frac{\bar{K}_2K_0-\bar{K}_1K_1}{\bar{K}_1\bar{K}_3-\bar{K}_2^2}\cos\varphi, \tag{1e,f}$$

In formulae (1) the following functions are denoted by the symbol K_i:

$$K_0(\beta)=\cosh\beta\,\cos\beta,\qquad K_1(\beta)=0.5[\cosh\beta\,\sin\beta+\sinh\beta\,\cos\beta],$$

$$K_2(\beta)=0.5\sinh\beta\,\sin\beta,\quad K_3(\beta)=0.25[\cosh\beta\,\sin\beta-\sinh\beta\,\cos\beta],$$

where: $\beta=x\sqrt[4]{3(1-\nu^2)}/\sqrt{rg}$, ν - Poisson's coefficient.
Overline denotes: $\bar{K}_i=K_i(x=l)$.

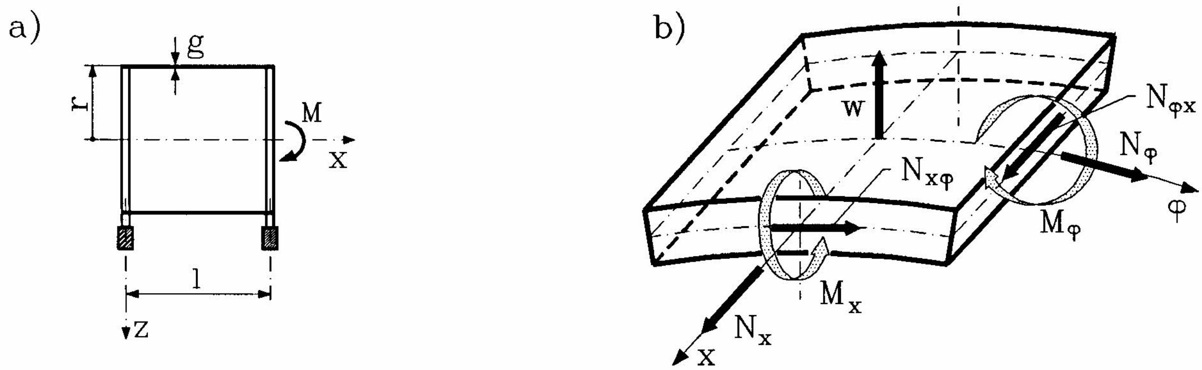

Fig. 3

2.3. Solution of a shell loaded transversely

In the case of a single-span shell shown in Fig. 1b the solution of general differential equations of the moment theory of reservoir shell without additional simplifying assumptions are given in [4]. External loads of the shell are expressed by double Fourier series:

$$p_\varphi=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}p_{\varphi mn}\sin m\varphi\cdot\sin\lambda\xi,\qquad p_r=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}p_{rmn}\cos m\varphi\cdot\sin\lambda\xi, \tag{2a,b}$$

where $\lambda=n\pi r/l$, $\xi=x/r$.

The coefficients of expansion (2) was expressed by the product of expansion coefficients $p_{\varphi n}, p_{rn}$ along axis x and expansion coefficients $p_{\varphi m}, p_{rm}$ around the perimeter:

$$p_{\varphi mn}=p_{\varphi n}\cdot p_{\varphi m},\qquad p_{rmn}=p_{rn}\cdot p_{rm} \tag{3a,b}$$

Radial displacement and shell forces are determined from the dependence:

$$w=\sum_m\sum_n w_{mn}\cos m\varphi\cdot\sin\lambda\xi,\quad N_x=\frac{D}{r}\sum_m\sum_n[-\lambda u_{mn}+(\nu+k\lambda^2)w_{mn}+\nu m u_{mn}]\cos m\varphi\cdot\sin\lambda\xi,$$

$$N_\varphi=\frac{D}{r}\sum_m\sum_n[m v_{mn}+(1+k-km^2)w_{mn}-\nu\lambda u_{mn}]\cos m\varphi\cdot\sin\lambda\xi, \tag{4a,b,c}$$

$$M_\varphi=-\frac{K}{r^2}\sum_m\sum_n[(m^2-1+\nu\lambda^2)w_{mn}]\cos m\varphi\cdot\sin\lambda\xi, \tag{4d}$$

$$M_x=-\frac{K}{r^2}\sum_m\sum_n[\nu m v_{mn}+(\nu m^2+\lambda^2)w_{mn}-\lambda u_{mn}]\cos m\varphi\cdot\sin\lambda\xi, \tag{4e}$$

where: $D=Eg/(1-\nu^2)$, $K=Eg^3/[12(1-\nu^2)]$, $k=K/(Dr^2)$, E - Young modulus.

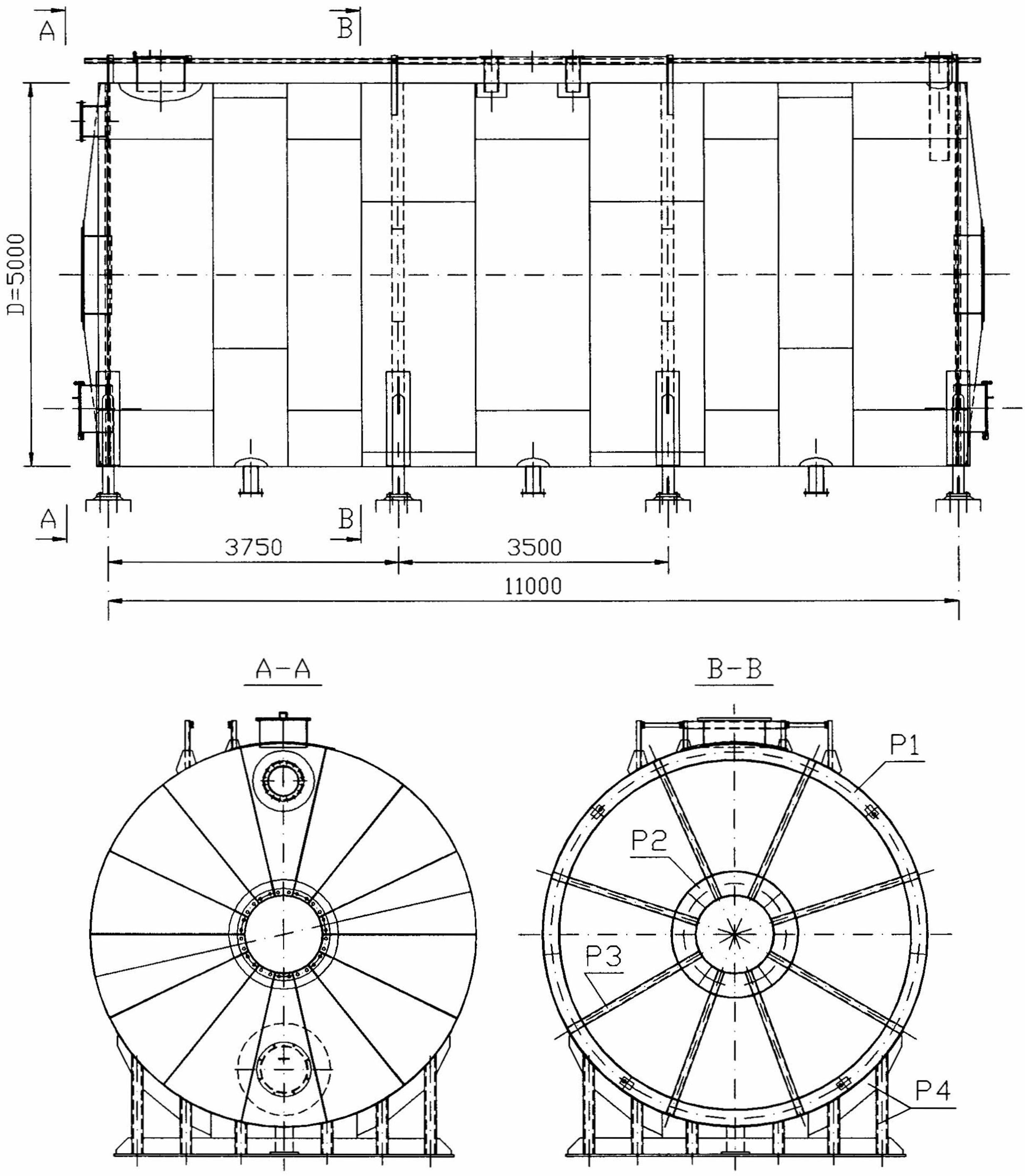

Fig. 4

Coefficients of development u_{mn}, v_{mn}, w_{mn} were determined from the following system of algebraic equations:

$$\left[\lambda^2+\frac{1-\nu}{2}m^2(1+k)\right]u_{mn}+\left(-\frac{1+\nu}{2}\lambda m\right)v_{mn}+\left[-\nu\lambda-k\left(\lambda^3\quad\frac{1-\nu}{2}\lambda m^2\right)\right]w_{mn}=0 \qquad (5a)$$

$$\left[-\frac{1+\nu}{2}\lambda m\right]u_{mn}+\left[m^2+\frac{1-\nu}{2}\lambda^2(1+3k)\right]v_{mn}+\left(m+\frac{3-\nu}{2}k\lambda^2 m\right)w_{mn}=\frac{r^2}{D}P_{\varphi mn}\,, \qquad (5b)$$

$$\left[-\nu\lambda-k(\lambda^3-\frac{1-\nu}{2}\lambda m^2)\right]u_{mn}+\left(m+\frac{3-\nu}{2}k\lambda^2 m\right)v_{mn}+[1+k(\lambda^4+2\lambda^2m^2+m^4-2m^2+1]w_{mn}=\frac{r^2}{D}P_{rmn} \tag{5c}$$

3. Local stability of the shell wall

Local stability of the shell wall was taken into account at the stage of dimensioning including complex instability states : 1) $N_x<0$, $N_\varphi<0$, 2) $N_x<0$, $N_\varphi>0$, 3) $N_x>0$, $N_\varphi<0$. Stability coefficients were determined in a way adjusted to the method of limit states according to [6].

4. Example

In Fig.4 is shown a prototype cylindrical reservoir of the volume $200m^3$ for production of biogas, of the wall thickness g=6mm. The reservoir was calculated and dimensioned by means of the authors' own computer program constructed in the language **C** according to a conception described above.

Of interest are frames made of an external ring **P1** connected with the internal ring **P2** by pipe rods **P3**. The reservoir is supported on openwork supports **P4**.

5. Remarks and Conclusions

1. The presented calculation method can be used to design all horizontal cylindrical reservoirs. In the case of reservoirs characterized by index $gl^2/r^3 < 10$, calculations should be made taking into consideration deformability of the cross-section and the influence of the bending state.

2. A characteristic feature of the constructed program is a short calculation time and high accuracy in comparison with the finite element method (on account of a relatively strong convergence of the applied series).

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